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Large Space Structure Integrated Structural and Control Optimization, Using Analytical Sensitivity Analysis

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The analytical sensitivity analysis procedure to implement the integrated structural and control optimization of a large, low-Earth-orbit space structure is dealt with. The objective is to show that significant computation time can be saved when the analytical procedure is used instead of numerical calculation of the sensitivity. Despite that the numerical approach is attractive because it does not require the hard algebraic task to develop the sensitivity derivative expressions, the approach is often responsible for the major computational cost of an optimization process. The analytical approach represents an attractive option when the computational effort becomes prohibitive under cost considerations.

Introduction

OPTIMIZATION plays an important role in a large field of applications. In the space area, the optimization is a consequence of the need to reach the best compromise between weight, volume, flexibility, stiffness, energy, and time, among other constraints. Launcher capabilities usually impose constraints on the weight, on the stiffness, and on the available volume for spacecraft. Because of the strong level of acceleration during the launching phase, the spacecraft must be stowed as rigidly as possible. Once in orbit, it is deployed and, depending on the vehicle elastic characteristics, the control structure interaction phenomenon may occur, risking the mission. Energy is another aspect of vital importance for spacecraft lifetime. The solution to these sorts of problems is optimization.

Sometimes we look for contradictory objectives like minimum energy and best performance during a maneuver, minimum weight, and maximum stiffness. The optimization procedures have to treat these contradictions by minimizing their effects (if any). The optimization approach should combine the needs, the availability, and the safety aspects of the whole project. The traditional approach to the design of a structure and control system has been sequential. The structure is first designed to satisfy the constraints on stresses, displacements, frequencies, and so on. Then the control system is designed for the given structure, aiming its orientation, guidance, and/or its motion to attain the required performance. If the nominal structure does not adhere to a satisfactory control design, it is returned to the structural group for modification. After modification, it is sent back to the control group for redesign. This separate-discipline approach has been used successfully in the past and is still used in cases where a nearly high-stiffness structure is reachable and where nonstructural components are the concentrated mass and inertia, or where performance requirements are not restrictive.

However, large space structures (LSS) and performance requirements do not meet this category, that is, the sequential approach generally does not lead to an optimum performance of the structure and control simultaneously. The problem cannot be solved efficiently by the control or structural designers acting separately. The interface between both areas becomes very important in achieving a compromise between the control and structural requirements. To accomplish this goal, we have implemented an integrated approach to optimize the structure and the control of an LSS. The approach consists of implementing the traditional sequential method in an integrated way.

Restrictions from both the structure and the control are set a priori, and the optimization process must satisfy those restrictions. It means that an interaction between the control and the structures groups is necessary, and the designer must know the structure and control areas to formulate the problem, taking into account the requirements from both areas. It is also necessary to integrate software to solve the optimization problem. We do not have specific software packages to solve integrated structural/control optimization problems. What we have are control optimization and structural optimization (different) packages. If the codes are compatible, then we can integrate the packages and solve the problem. In the study presented in this paper we have integrated NEWSUMT-A and ORACLS software packages to solve the structural/control optimization problem. After obtaining the optimal structure and the optimal control we used the MATLAB® software to solve the transient phase regarding the attitude and the vibration control.

However, the main goal of this work is to show the integrated structural/control optimization study by the use of semi-analytical computation of the sensitivity of eigenvalue and control damping, imposed as constraints in the problem formulation. The main motivation to treat the subject of sensitivity in the scope of this paper is that it plays a very important role in the structural optimization area. The sensitivity of the structural response to changes in the design variables is frequently the major computational cost of the optimization process. From the control theory viewpoint, sensitivity is concerned with the variations in the control index caused by variations in the plant and control influence matrices. One option to overcome the highly expensive computational time associated with the sensitivity numerical calculation is the derivation of analytical expressions for the sensitivity analysis. The objective of this paper is to present an integrated structural/control optimization problem formulation by using analytical expressions for the sensitivity calculation and to enhance the computational savings time of the process compared to the integrated structural/control procedure. This will be accomplished by the use of numerical sensitivity calculation via finite differences. Figure 1 shows a flow diagram of the approach[1] we have used to implement the integrated structural/control optimization.

Problem Formulation: Equations of Dynamics

Figure 2 shows the physical model of a low-Earth-orbit (LEO) LSS subject to the gravity-gradient torque that is the focus of this paper. The LSS is assumed to be a large, tubular, one-dimensional

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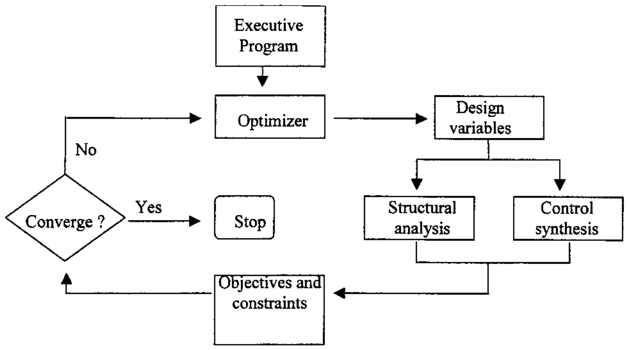

Fig. 1 Integrated structural/control optimization flow diagram.

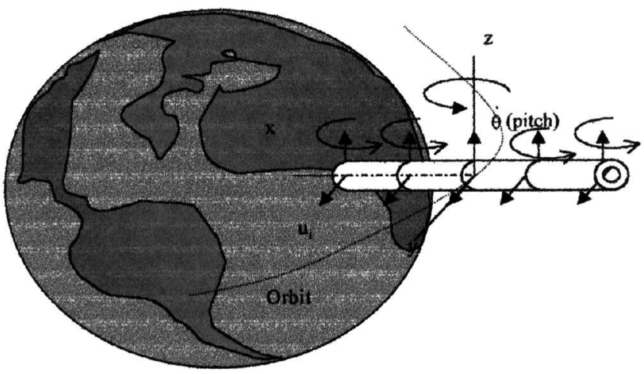

Fig. 2 LEO LSS in-orbit configuration.

beam. The attitude motion (pitch) and the elastic vibration are constrained to the orbital plane. The Lagrangian formulation is combined with the finite element technique to derive the dynamics equations of motion. The structure has been divided into four finite elements for modeling purposes. By using the finite elements method in conjunction with the Lagrangian formulation, we can write the equations of motion as

$$M\ddot{q} + Sq - \frac{1}{2}\frac{\partial}{\partial q}(\dot{q}^T M \dot{q}) + \frac{dM}{dt}\dot{q} = Q_q \tag{1}$$

where M and S are the mass and the rigidity matrices, respectively. S includes the contribution of the gravitational potential energy, and q is the generalized coordinate associated with the system degrees of freedom. Q_q is the generalized control force.

Let us assume that the nominal equilibrium state is described in attitude by the gravity-gradient stabilized configuration (longitudinal axis aligned with the local vertical) and in elastic vibration by a closely nondeformed structure configuration. By linearizing the equations of motion about such a state, we can write the associated linear system equation as

$$M\ddot{q} + Sq = Q_q \tag{2}$$

Analytical Sensitivity Expressions Derivation

The simplest technique for calculating derivatives of response with respect to a design variable is the finite difference approximation. This technique is often computationally expensive, and finite difference approximations often have inaccuracy problems. Despite that the numerical approach is attractive because it does not require the difficult algebraic task of developing the sensitivity derivative expressions, the approach is often responsible for the major computational cost of an optimization process. The analytical approach represents an attractive option when the computational effort becomes prohibitive under cost considerations.

In general, when the integrated structural/control problems are solved, a significant part of the dynamics analysis involves the eigensolution of the problem

$$(S - \omega^2 M)\phi = 0 \tag{3}$$

where ω^2 is the diagonal matrix of the squared structural frequencies. If the constraint is imposed on a frequency, as it is the case of the present study, we have to find the derivative of the structural frequency. The solution requires solving the associated eigenvalue problem. Even when analytical expressions for sensitivity analysis are derived, a challenging task still remains for the computer work. Only for very low-order and simple problems is it possible to obtain the analytical sensitivity expressions explicitly for the eigenvalues and eigenvectors. In other words, if we have a high-order problem, it is not possible to obtain analytical expressions for the eigenvalues and the eigenvectors in terms of the design variables. Systems with large numbers of degrees of freedom characterize the LSS models. By virtue of this, the problem of using analytical expressions for sensitivity analysis still involves numerical calculation, such as the eigenvalue problem solution and eigenvector computation. However, even when we have to solve the eigenvalue problem via computer, the computational effort is less than that when we have to compute the derivatives by using, for example, a finite differences procedure. To restate, consider the derivative of the eigenvalue problem given by Eq. (1) with ω^2 replaced by λ and assume S and M to be symmetric. The derivative of the eigenvalue problem[2] can be written as follows:

$$(S - \lambda_j M)\phi_j' + (S' - \lambda_j M')\phi_j - \lambda_j' M\phi_j = 0 \tag{4}$$

where prime denotes the partial derivative with respect to the design variable X_j. By adopting the normalization with respect to the mass matrix, we have

$$\phi_j^T M \phi_j = 1 \tag{5}$$

By transposing Eq. (3) and substituting the result into Eq. (4) premultiplied by ϕ^T, we obtain the eigenvalue derivatives with respect to the design variables:

$$\lambda_j' = \frac{\phi_j^T(S' - \lambda_j M')\phi_j}{\phi_j^T M \phi_j} \tag{6}$$

In view of the normalization condition given by Eq. (5), Eq. (6) can be rewritten as

$$\lambda_j' = \phi_j^T(S' - \lambda_j M')\phi_j \tag{7}$$

Note that $\lambda_j = \omega_j^2$. We can obtain the frequency derivative by exploiting the λ_j derivative

$$\lambda_j' = \frac{\partial \omega_j^2}{\partial X_j} = 2\omega_j \frac{\partial \omega_j}{\partial X_j} \tag{8}$$

By using Eq. (7), we obtain

$$2\omega_j \frac{\partial \omega_j}{\partial X_j} = \phi_j^T\left(S' - \omega_j^2 M'\right)\phi_j \tag{9}$$

or

$$\omega_j' = \frac{\phi_j^T\left(S' - \omega_j^2 M'\right)\phi_j}{2\omega_j} \tag{10}$$

Note that the resulting equations are functions of the eigenvalue λ_j and the eigenvector ϕ_j so that the need to solve the eigenvalue problem still remains. The approach used here consists of computing the partial derivatives of M with respect to the design variables and then using the expression given by Eq. (6) [or, alternatively, Eq. (10)] to compute the eigenvalue sensitivity. It is not that difficult to obtain the derivatives of matrices M and S with respect to the design variable A_i, the cross-sectional area of the tubular beam. We have fixed the structure length so that the only design variable is the cross-sectional area. Note that the moments of inertia of area that appear in the stiffness matrix S can be written also in terms of the cross-sectional areas. One may want to use the thickness or the tube diameter as a design variable. Another simplification we have adopted here regards the material density that we have assumed to be constant. Note that the eigenvalue problem solution is required anyway if a constraint is imposed on a frequency of vibration. The merit of obtaining the analytical sensitivity is that its computation becomes just a problem of solving an algebraic equation provided that the partial derivatives of matrix $\boldsymbol{M}$ is given analytically. If a constraint is also imposed on an eigenvector, then we have to compute the eigenvector derivatives with respect to the design variables. The eigenvector derivative can be obtained by taking the derivative of Eq. (5) with respect to the design variables:

$$\left(\phi_j^T\right)'\boldsymbol{M}\phi_j + \phi_j^T\boldsymbol{M}'\phi_j + \phi_j^T\boldsymbol{M}\phi_j' = 2\phi_j^T\boldsymbol{M}\phi_j' + \phi_j^T\boldsymbol{M}'\phi_j = 0$$

$$\phi_j^T\boldsymbol{M}\phi_j' = -\tfrac{1}{2}\phi_j^T\boldsymbol{M}'\phi_j \tag{11}$$

Equation (4) and the second of Eqs. (11) can be arranged to yield the eigenvector derivatives directly as[2]

$$\begin{bmatrix} \boldsymbol{S}-\lambda_j\boldsymbol{M} & -\boldsymbol{M}\phi_j \\ -\phi_j^T\boldsymbol{M} & \boldsymbol{0} \end{bmatrix}\begin{Bmatrix} \phi_j' \\ \lambda_j' \end{Bmatrix} = \begin{Bmatrix} -(\boldsymbol{S}'-\lambda_j\boldsymbol{M}')\phi_j \\ \tfrac{1}{2}\phi_j^T\boldsymbol{M}'\phi_j \end{Bmatrix} \tag{12}$$

We have to be very careful when using this equation because the principal minor $(\boldsymbol{S}-\lambda_j\boldsymbol{M})$ is singular. The approximate computation of the eigenvector derivatives constitutes one of the techniques to overcome the problem.[3] One approximation of the eigenvector derivative is

$$\phi_j' = \sum_{k=1}^{n}\alpha_{ijk}\phi_k \tag{13}$$

The expression for α_{ijj} $(k = j)$ is obtained by substituting Eq. (13) into the second of Eqs. (11), as follows:

$$\sum_{k=1}^{n}\alpha_{ijk}\phi_j^T\boldsymbol{M}\phi_k = -\frac{1}{2}\phi_j^T\boldsymbol{M}'\phi_j \tag{14}$$

Then, for $k = j$, we have, after taking into account the normalization with respect to the mass matrix relationship given by Eq. (5),

$$\alpha_{ijj} = -\tfrac{1}{2}\phi_j^T\boldsymbol{M}'\phi_i \tag{15}$$

The coefficient α_{ijj} (for $k = l \neq j$) can be computed by substituting Eqs. (15) into Eq. (4), premultiplying the resulting equation by ϕ_l^T, and using the approximation given by Eq. (15),

$$\sum_{k=1}^{n}\alpha_{ijk}\phi_l^T(\boldsymbol{S}-\lambda_j\boldsymbol{M})\phi_k + \phi_l^T(\boldsymbol{S}'-\lambda_j\boldsymbol{M}')\phi_i - \lambda_j'\phi_l^T\boldsymbol{M}\phi_i = 0 \tag{16}$$

Because the eigenvectors are $\boldsymbol{M}$ orthogonal, the last term in this equation is zero. Exploiting this expression for $k = l \neq j$, we have the expressions $\phi_l^T\boldsymbol{S}\phi_l$ and $\phi_l^T\boldsymbol{M}\phi_l$. From the eigenvalue problem, we can rewrite these expressions as, respectively,

$$\phi_l^T\boldsymbol{S}\phi_l = \lambda_l\phi_l^T\boldsymbol{M}\phi_l, \qquad \phi_l^T\boldsymbol{M}\phi_l = 1$$

By substituting these expressions into Eq. (16), we obtain

$$\alpha_{ijl} = \frac{\phi_l^T(\boldsymbol{S}'-\lambda_j\boldsymbol{M}')\phi_i}{(\lambda_j-\lambda_i)}, \qquad l \neq j \tag{17}$$

Observe here once again that to solve this equation and to obtain the eigenvector derivative we need to solve the eigenvalue problem and to compute the eigenvectors. This can be done analytically only for simple and low-order systems.

Now let us focus on the control aspects of our problem regarding sensitivity. Consider the linear quadratic regulator problem associated with the control law to perform the structural vibration suppression and the attitude control. The equation of the dynamics associated with the eigenvalue sensitivity problem discussed earlier is

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{S}\boldsymbol{q} = \boldsymbol{D}\boldsymbol{f} \tag{18}$$

where $\boldsymbol{D}$ is the distribution matrix that relates the control input vector $\boldsymbol{f}$ to the coordinate system. The system equation in the state space is given by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f} \tag{19}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{S} & \boldsymbol{0} \end{bmatrix} \tag{20}$$

is the open-loop matrix and

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{D} \end{bmatrix} \tag{21}$$

$$\boldsymbol{x} = \begin{Bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{Bmatrix} \tag{22}$$

To design a controller with the liner quadratic regulator, the control optimization problem can be stated in terms of the assumed known state[4,5] by minimizing the index

$$J = \int_0^{\infty}(\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{f}^T\boldsymbol{R}\boldsymbol{f})\,\mathrm{d}t \tag{23}$$

subject to Eq. (19), where $\boldsymbol{Q}$ is the state weighting matrix that has to be positive semidefinite and $\boldsymbol{R}$ the control weighting matrix that has to be positive definite. The result of minimizing the quadratic performance index and satisfying the state equation yields the state feedback control law

$$\boldsymbol{f} = -\boldsymbol{F}\boldsymbol{x} \tag{24}$$

where $\boldsymbol{F}$ is the optimum gain matrix given by

$$\boldsymbol{F} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} \tag{25}$$

and where $\boldsymbol{P}$ is a symmetric positive definite matrix called the Riccati matrix. $\boldsymbol{P}$ is found by solving the algebraic Riccati equation[5]:

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} + \boldsymbol{Q} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} = \boldsymbol{0} \tag{26}$$

By substituting Eq. (24) into Eq. (19), we obtain the governing equations for the optimal closed-loop system as

$$\dot{\boldsymbol{x}} = \boldsymbol{C}\boldsymbol{x} \tag{27}$$

where

$$\boldsymbol{C} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{F} \tag{28}$$

$\boldsymbol{C}$ is the closed-loop matrix for this application (sometimes $\boldsymbol{C}$ is also called the observation matrix when observer dynamics are also included).

Next we give the analytical procedure to obtain derivatives of the eigenvalues associated with the closed-loop matrix $\boldsymbol{C}$. Note that $\boldsymbol{C}$ involves not only the open-loop matrix $\boldsymbol{A}$ and the input matrix $\boldsymbol{B}$, but also the Riccati matrix $\boldsymbol{P}$ [through $\boldsymbol{F}$, see Eq. (26)]. We can derive the expression for the eigenvalue derivative associated with $\boldsymbol{C}$ in a way similar to how we derived the derivative of the eigenvalue given in Eq. (3). The expression for the eigenvalue derivative we present here follows closely the derivation presented in Ref. 6. Let μ_j be the eigenvalues of matrix $\boldsymbol{C}$ that can be written as

$$(\boldsymbol{C} - \mu_j\boldsymbol{I})\alpha_j = 0 \tag{29}$$

$$\beta_j^T(\boldsymbol{C} - \mu_j\boldsymbol{I}) = 0 \tag{30}$$

where β_j and α_j represent the left and right eigenvectors of $\boldsymbol{C}$, respectively.

By differentiating Eq. (29) with respect to the design variable and premultiplying by β_j^T, we have

$$\beta_j^T(\boldsymbol{C}' - \mu_j I)\alpha_j + \beta_j^T(\boldsymbol{C} - \mu_j\boldsymbol{I})\alpha_j' = 0 \tag{31}$$

By using Eq. (30), we obtain

$$\mu_j' = \frac{\phi_j^T\boldsymbol{C}'\alpha_j}{\beta_j^T\alpha_j} \tag{32}$$

Let

$$\beta_j^T a_j = c \tag{33}$$

where c is constant and given by a biorthogonality normalization condition. This constant is assumed to equal unity in general. Before working on $\boldsymbol{C}'$, let us write it in terms of the Riccati matrix $\boldsymbol{P}$ so that we can see clearly that the $\boldsymbol{C}$ derivative with respect to a design variable implies differentiating the algebraic Riccati matrix with respect to that variable. The matrix $\boldsymbol{C}$ in terms of the Riccati matrix $\boldsymbol{P}$ can be written by substituting Eq. (25) into Eq. (28) to yield

$$\boldsymbol{C} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} \tag{34}$$

Then,

$$\boldsymbol{C}' = \boldsymbol{A}' - \boldsymbol{G}'\boldsymbol{P} - \boldsymbol{G}\boldsymbol{P}' \tag{35}$$

where

$$\boldsymbol{G} = \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T \tag{36}$$

The derivative of the state matrix $\boldsymbol{A}$ can be obtained through Eq. (20). The derivative of the input matrix $\boldsymbol{B}$ is obtained in terms of the derivative of the matrix $\boldsymbol{M}$ as shown in Eq. (21). We still need an expression for derivative of the Riccati matrix $\boldsymbol{P}$ to have the complete sensitivity of $\boldsymbol{C}$. To accomplish this, we turn back to the Riccati algebraic equation given by Eq. (26). By solving this equation for Eq. (32) in conjunction with Eq. (31), we obtain

$$\boldsymbol{C}^T\boldsymbol{P} + \boldsymbol{P}^T\boldsymbol{G}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{C} + \boldsymbol{Q} = 0 \tag{37}$$

By taking the derivative of each term in Eq. (37) with respect to the design variable, we obtain

$$\boldsymbol{C}'^T\boldsymbol{P} + \boldsymbol{C}^T\boldsymbol{P}' + \boldsymbol{P}'\boldsymbol{C} + \boldsymbol{P}\boldsymbol{C}' + \boldsymbol{P}'^T\boldsymbol{G}^T\boldsymbol{P} + \boldsymbol{P}^T\boldsymbol{G}'^T\boldsymbol{P} + \boldsymbol{P}^T\boldsymbol{G}^T\boldsymbol{P}' = 0 \tag{38}$$

Note that the matrix $\boldsymbol{Q}$ is constant, and its derivative with respect to the design variables is zero.

By substituting Eq. (35) into Eq. (38), we obtain the Lyapunov equation[7]:

$$\boldsymbol{C}^T\boldsymbol{P}' + \boldsymbol{P}'\boldsymbol{C} = \tilde{\boldsymbol{C}} \tag{39}$$

where $\tilde{\boldsymbol{C}} = -(\boldsymbol{A}')^T\boldsymbol{P} - \boldsymbol{P}\boldsymbol{A}' + \boldsymbol{P}\boldsymbol{G}'\boldsymbol{P}$. The solution of Eq. (39) yields the sensitivity of the Riccati matrix $\boldsymbol{P}$, $\boldsymbol{P}'$, with respect to the design variable accounted for in the problem. Because the sensitivities of the frequencies μ_j are known, the damping sensitivity can be computed by differentiating the closed-loop damping factor ξ with respect to the design variable. The closed-loop eigenvalue can be written as

$$\boldsymbol{\mu}_i = \sigma_i \pm j\varpi_i \tag{40}$$

and its modulus can be written as

$$\mu_i = \sqrt{\sigma_i^2 + \varpi_i^2} \tag{41}$$

Here, $j = \sqrt{(-1)}$.

The closed-loop damping factor associated with the control is

$$\xi_i = \sigma \Big/ \sqrt{\sigma_i^2 + \varpi_i^2} \tag{42}$$

By differentiating Eq. (42) with respect to a design variable, we obtain

$$\xi_i' = (\sigma_i'\mu_i - \mu_i'\sigma_i) \big/ \mu_i^2 \tag{43}$$

This expression can be written in terms of the derivatives of ω and σ by using Eq. (42) to obtain

$$\xi_i' = \frac{\omega_i(\sigma_i'\omega_i - \sigma_i\omega_i')}{\left(\sigma_i^2 + \omega_i^2\right)^{\frac{3}{2}}}$$

ω_i' can be obtained semi-analytically through Eq. (10). It is not possible to obtain the whole damping factor derivative analytically for a high-order system. Equation (43) yields the sensitivity of the control damping parameter.

Integrated Optimization Problem Formulation

The linear quadratic regulator has been used for the pitch and vibration control according to Eq. (23). The structural optimization implemented here can be stated by[8] minimizing the structural weight

$$W = \sum_{i=1}^{n_e} \rho_i A_i \ell_i \tag{44}$$

subject to constraints such as

$$g_j(W) \geq 0, \qquad g_j(\omega) \geq 0, \qquad g_j(\xi) \geq 0 \tag{45}$$

where ξ, ω, and W are the control damping factor, the frequency of vibration, and the structure weight, respectively. Side constraints were also imposed on the structure cross-sectional areas. The side constraints ensure that the design fits to the realistic characteristics of the project.

The integrated approach implemented in this study is somewhat sequential in the sense that the control and the structure are optimized sequentially. However, the simulation stops when a compromise between the control optimization and the structural optimization is reached according to the constraints imposed on the problem. This compromise is represented by the convergence of the solution satisfying the constraints involving both the control and the structure requirements.

Computer Simulations and Results

The computer simulations have shown that the savings in computer time (CPU time) is about 50% when we use the analytical expressions for the sensitivity calculation. This result makes sense only for the model treated here. The system we have implemented in the computer is of the order 22×22 because we have divided our structure into only four finite elements. This refers to state-space equations (including the pitch degree of freedom). The CPU is the personal computer's brain. The CPU time is the time the computer is involved in processing some task. It differs from the simulation elapsed time (clock time) and is not the time the computer was assigned to solve a specific task. Personal computers process sequentially. This means that a large CPU time makes the computer simulation very expensive. For the model focused on in this study, the CPU time is about 10 min when the semi-analytical approach is used, whereas for the pure numerical approach the CPU time is about 20 min. The CPU time may differ either if the output data are large or the criteria for convergence are very stringent.

For illustration of one case, we have set the convergence criteria for the unconstrained minimization to be of the order of $10E{-}10$ and the CPU time increased to almost one day compared to the same problem with the criteria set to $10E{-}5$. In general, finite element models take into account many more elements, and the number of state equations is much higher than the model treated here. We did not vary the number of finite elements to check on how the CPU time increases with the system degrees of freedom enlargement. However, it is intuitive that CPU time increases greatly when the order of the system is augmented. The difference between the optimization results when using finite differences and using the analytical expressions for the sensitivity computation are practically the same for the problem studied here. This means that there is no problem regarding the accuracy when we use the finite difference approach to compute the sensitivity derivatives. The main result is the time savings associated with the computer simulations to implement the integrated structural optimization procedure. Figures 3–6 show the results of the integrated structural optimization for the LSS studied in the paper. The weight savings resulting from the integrated optimization is relevant, (about 20%). The result regarding the control performance is better for the optimized structure when compared to the initial given structure.

Three actuators have been used (see Fig. 3 for illustration): two force actuators (at the tips of the structure) and one torque actuator (at the middle of the structure). The input data for the simulations are $E = 7.3084 \times 10^{10}$ N/m^2; $\rho = 2768$ kg/m^3; $L = 250.88$ m; $A_i = 3.24 \times 10^{-3}$ m^2; $h = 483.77$ km; $\mu_\oplus = 3.986 \times 10^5$ km^3/s^2; $\theta_0 = 6$ deg, $\nu = 2.5$ m; $\omega \geq 5.2515e{-}002$ Hz; $\xi \geq 0.27$; $Q(1, 1) = 10^5$;

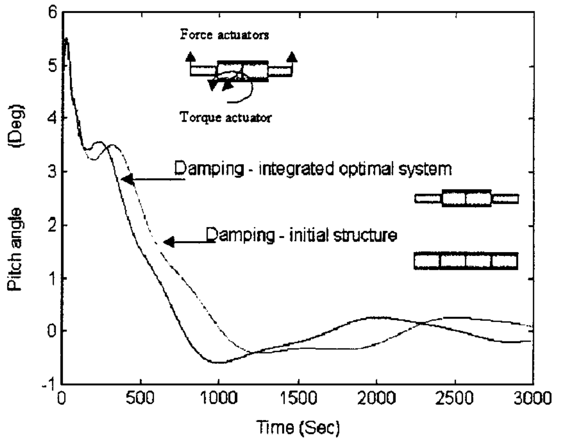

Fig. 3 Pitch angle control for both nominal and optimized structure.

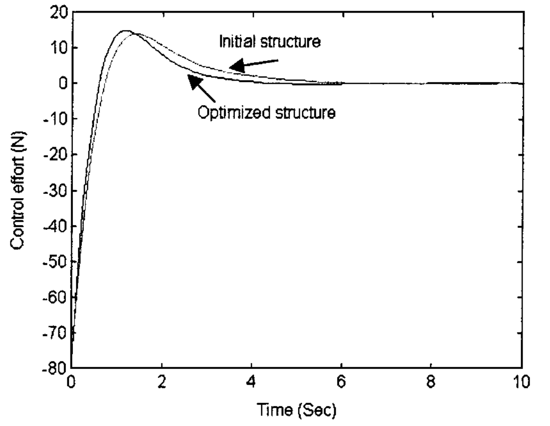

Fig. 6 Control effort to damp vibration at tip of structure for both nominal and optimized structure.

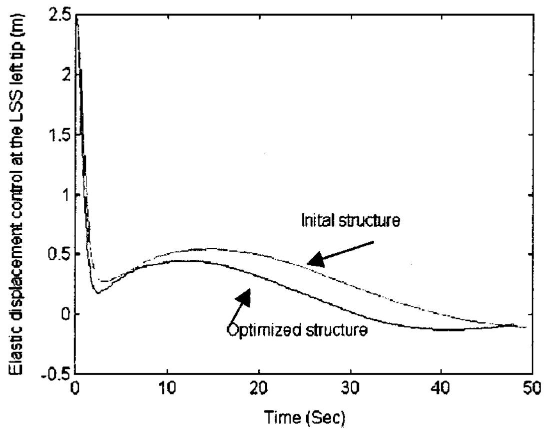

Fig. 4 Damping of elastic displacement, tip of LSS.

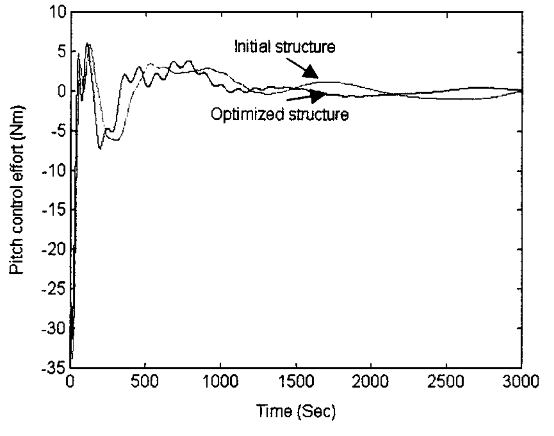

Fig. 5 Pitch control effort for both nominal and optimized structure.

$Q(i, j) = 10^3$, $i = j = 1, 2, \ldots, 10$; $Q(i, j) = 1$, $i = j = 12, 13, \ldots, 22$; $Q(i, j) = 0$ for $i \neq j$.

The side constraints are $1.62 \leq X_i \leq 3.24 \times 10^{-3}$ m^2 ($i = 1, 2, 3, 4$). $E, \rho, L, A_i, h, \mu_{\oplus}, \theta_0$, and ν are the Young's modulus, the material density, the structure length, the structure cross-sectional areas, the orbit altitude, the gravitational constant, the initial pitch angle, and the structure tip displacement, respectively. The optimal structure is given by the cross-sectional areas: $A_1 = A_3 = 1.62 \times 10^{-3}$ m^2 and $A_2 = A_3 = 3.24 \times 10^{-3}$ m^2. The weight savings by the integrated optimization process is about 20% with respect to the initial structural weight.

Figure 4 shows the vibration damping of the elastic displacement at the left tip of the structure. The better control performance is not very evident during the first 7 s. It becomes more apparent when the displacement approaches to zero.

Figure 5 shows the control effort for both the nominal and the optimized structure. Once again, the difference of the control efforts for both the given structure and the optimized one becomes more apparent when the efforts approach zero. For the case of the optimized structure, the control efforts approach zero more rapidly than they do for the given structure.

Figure 6 shows the control effort to damp the vibration at the tip of the structure for both the given and the optimized structure. The result for the optimized structure is slightly better than that for the given structure, most of the time. However, we do not have a significant gain for the short phase when the control effort is large for both the given and the optimized structure.

Conclusions

The results have shown a significant time savings when analytical expressions are used to compute the sensitivity derivatives instead of the numerical finite difference method. This result is very important because the numerical methods may become prohibitive given cost considerations because of the excessive computational effort necessary to compute the sensitivity derivatives. Note that when solving the eigenvalue problem, we can use methods to find only those eigenvalues necessary to the problem solution, avoiding the computational effort involved in computing all of the eigenvalues and eigenvectors unnecessarily.

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